Machine Learning 1 – Exercise 4

Machine Learning for Computer Vision
TU Dresden

Solutions to any part of any exercise will be accepted as separate postings in the thread entitled **Exercise 4: Solution** of the lecture forum[1] until **Jan 4th, 18:00**. The solutions will not be graded. At the end of this term, the most highly voted solution will be awarded with a book prize.

1 Clustering

a) Show that it is sufficient in (7.1) of the lecture notes[2] to consider only chordless cycles.

b) How many chordless cycles are there in a complete graph with $n \in \mathbb{N}$ nodes?

c) Show: For the special case of complete graphs, the graph decomposition problem specializes to the set partition problem.

d) Define procedures for computing the following differences in cost efficiently:

i) $\varphi(y^{\text{join}_{BC}[\Pi]}) - \varphi(y^{\Pi})$, cf. Algorithm 4 in the lecture notes[2]

ii) $\varphi(y^{\text{move}_{aU}[\Pi]}) - \varphi(y^{\Pi})$, cf. Algorithm 5 in the lecture notes[2]

2 Ordering

a) Define a procedure for computing the difference $\varphi(y^{\text{transpose}_{jk}[\alpha]}) - \varphi(y^{\alpha})$ in Algorithm 7 of the lecture notes efficiently.

b) As an example of an ordering problem with known costs, consider the season ranking of teams in a sports league in which every team competes against every other team twice (home and away) in a season. Consider two mathematical abstractions of the ranking problem:

- Scores are assigned to teams based on the outcomes of the individual competitions: a win is worth 3 points, a tie is worth 1 point, and a loss is worth 0 points. A total preorder of the teams is defined as the total preorder of their respective total scores.
- A solution to the linear ordering problem with costs defined as follows. For every individual competition (a, b) of Team a (home) versus Team b (away), let $c_{ab} = -1$ in case a wins, $c_{ab} = 1$ in case b wins, and $c_{ab} = 0$ in case of a tie.

Give an example of outcomes of a season's games in which the two abstractions lead to different rankings.

Explain informally how the two systems differ. Are all games equally important?

[1] `https://bildungsportal.sachsen.de/opal/auth/RepositoryEntry/26617479170/CourseNode/102502724177602`

[2] `https://mlcv.inf.tu-dresden.de/courses/wt20/ml1/ml1-lecture-notes.pdf`