Machine Learning 1 – Exercise 3

Machine Learning for Computer Vision
TU Dresden

Solutions to any part of any exercise will be accepted as separate postings in the thread entitled **Exercise 3: Solution** of the lecture forum[1] until **Dec 7th, 18:00**. The solutions will not be graded. At the end of this term, the most highly voted solution will be awarded with a book prize.

1 Partitioning

a) For each partition Π of the set $A = \{a, b, c\}$, write down:

   i) the equivalence relation $\equiv_\Pi$ induced by the partition

   ii) the binary labeling y^Π defined in (6.2) of the lecture notes[2]

b) Prove, for any finite set A, that the map $\Pi \mapsto y^\Pi$ from partitions to binary labelings is a bijection from the set of all partitions of A to the set

$$\mathcal{Y} = \left\{ y : \tbinom{A}{2} \to \{0,1\} \;\middle|\; \forall a \in A \; \forall b \in A \setminus \{a\} \; \forall c \in A \setminus \{a,b\} \colon \; y_{\{a,b\}} + y_{\{b,c\}} - 1 \leq y_{\{a,c\}} \right\}$$

c) Define an instance of the set partition problem for which the output of the greedy joining algorithm, initialized with singleton subsets, is strictly improved by greedy moving.

d) Define procedures for computing the following differences in cost efficiently:

   i) $\varphi(y^{\mathrm{join}_{BC}[\Pi]}) - \varphi(y^\Pi)$, cf. Algorithm 1 in the lecture notes[2]

   ii) $\varphi(y^{\mathrm{move}_{aU}[\Pi]}) - \varphi(y^\Pi)$, cf. Algorithm 2 in the lecture notes[2]

e) Define a new local search algorithm for the set partition problem by applying the technique of Kernighan and Lin to greedy *joining*.

f) If both algorithms are initialized with singleton sets, can greedy joining with the technique of Kernighan and Lin converge to feasible solutions with strictly lower cost than those found by greedy joining without the technique of Kernigan and Lin?

[1] `https://bildungsportal.sachsen.de/opal/auth/RepositoryEntry/26617479170/CourseNode/102502724177602`

[2] `https://mlcv.inf.tu-dresden.de/courses/wt20/ml1/ml1-lecture-notes.pdf`