Machine Learning II

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Machine Learning for Computer Vision
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https://mlcv.cs.tu-dresden.de/courses/25-summer/ml2/

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So far:

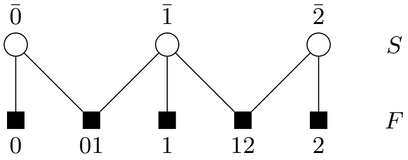
- Features $x_s \in X$ are defined for single samples $s \in S$ only.
- Dependencies between decisions $y_s, y_{s'} \in \{0,1\}$ for distinct $s, s' \in S$ are only due to hard constraints defined a feasible set $\mathcal{Y} \subset \{0,1\}^S$.

Next, we will define the **supervised structured learning** problem and the **structured inference** problem in which

- features are associated with subsets of S
- decisions can be tied by probabilistic dependencies.

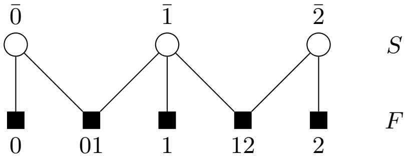
More specifically, we will

- introduce a family $H : \Theta \to \mathbb{R}^{X \times Y}$ of functions that quantify by $H_\theta(x, y)$ how incompatible features $x \in X$ are with a combination of decisions $y \in \{0,1\}^S$
- define supervised structured learning as a problem of finding one function from this family
- define structured inference as the problem of finding a combination of decisions $y \in \{0,1\}^S$ that minimizes $H_\theta(x, \cdot)$.



Definition. A triple (S, F, E) is called a **factor graph** with **variable nodes** S and **factor nodes** F iff $S \cap F = \emptyset$ and $(S \cup F, E)$ is a bipartite graph such that $\forall e \in E \, \exists s \in S \, \exists f \in F \colon e = \{s, f\}$.

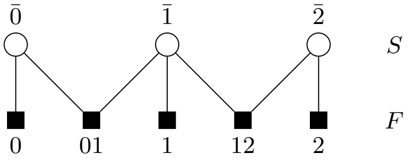
- For any factor node $f \in F$, we denote by $S_f = \{s \in S \mid \{s, f\} \in E\}$ the set of those variable nodes that are neighbors of f.
- For any variable node $s \in S$, we denote by $F_s = \{f \in F \mid \{s, f\} \in E\}$ the set of those factor nodes that are neighbors of s.



Definition. A tuple $T = (S, F, E, \{X_f\}_{f \in F}, x)$ is called **unlabeled structured data** iff the following conditions hold:

- (S, F, E) is a factor graph
- Every set X_f is non-empty, called the **feature space** of f
- $x \in \prod_{f \in F} X_f$, where the Cartesian product $\prod_{f \in F} X_f$ is called the **feature space** of T.

A tuple $(S, F, E, \{X_f\}_{f \in F}, x, y)$ is called **labeled structured data** iff $(S, F, E, \{X_f\}_{f \in F}, x)$ is unlabeled structured data, and $y \in \{0, 1\}^S$.



Definition. W.r.t. any labeled structured data $(S, F, E, \{X_f\}_{f \in F}, x, y)$,

- the feature space $X = \prod_{f \in F} X_f$
- the set $Y = \{0, 1\}^S$
- any $\Theta \neq \varnothing$ and family of functions $H : \Theta \to \mathbb{R}^{X \times Y}$
- any $R : \Theta \to \mathbb{R}_0^+$, called a **regularizer**
- any $L : \mathbb{R}^Y \times Y \to \mathbb{R}_0^+$, called a **loss function**
- any $\lambda \in \mathbb{R}_0^+$, called a **regularization parameter**,

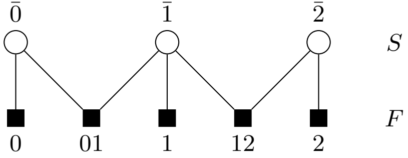
the instance of the **supervised structured learning problem** has the form

$$\inf_{\theta \in \Theta} \quad \lambda R(\theta) + L(H_\theta(x, \cdot), y) \tag{1}$$

Example.

$$L(H_\theta(x,\cdot),y) = H_\theta(x,y) + \ln \sum_{y' \in \{0,1\}^S} e^{-H_\theta(x,y')} \tag{2}$$

$$R(\theta) = \|\theta\|_2^2 \tag{3}$$



Definition. With respect to

- any unlabeled structured data $T = (S, F, E, \{X_f\}_{f \in F}, x)$
- any $\hat{H} \colon X \times \{0,1\}^S \to \mathbb{R}$

the instance of the **structured inference problem** has the form

$$\min_{y \in \{0,1\}^S} \quad \hat{H}(x, y) \tag{4}$$

Summary.

- **Structured data** consists of a factor graph (S, F, E) and features $x_f \in X_f$ for every factor $f \in F$.
- The **structured learning problem** is an optimization problem whose feasible solutions θ define functions $H_\theta : X \times Y \to \mathbb{R}$ whose values $H_\theta(x, y)$ quantify an incompatibility of features $x \in X$ and combinations of decisions $y \in \{0, 1\}^S$.
- The **structured inference problem** consists in finding decisions $y \in \{0, 1\}^S$ compatible with given features $x \in X$, by minimizing a given incompatibility function $\hat{H}(x, \cdot)$.