Machine Learning 1 – Exercise 1

Machine Learning for Computer Vision
TU Dresden

Deciding with disjunctive normal forms (DNFs)

a) Let $V = \{0, 1, 2, 3\}$. State the V-variate DNF defined by $\theta = \{(\emptyset, \{0\}), (\{0\}, \{3\}), (\{0, 3\}, \{1, 2\})\}$, its length and its depth.

b) State two distinct DNFs such that the function defined by these DNFs equals the function g defined in Tab. 1 below.

c) How many distinct DNFs in $n = |V|$ variables exist?

d) Prove the following universality property: For any finite, non-empty set V and any $f \colon \{0, 1\}^V \to \{0, 1\}$, there exists a V-variate DNF defining f.

Table 1: Defined by the value table below is a Boolean function $g \colon \{0, 1\}^V \to \{0, 1\}$ with $V = \{0, 1, 2\}$.

x_0	x_1	x_2	$g(x)$
0	0	0	0
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	1
1	0	1	0
0	1	1	1
1	1	1	0