Computer Vision I

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Machine Learning for Computer Vision
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For any $n \in \mathbb{N}$, let $[n] := \{0, \ldots, n-1\}$.

Definition 1. A **digital image** of **width** $n_0 \in \mathbb{N}$ and **height** $n_1 \in \mathbb{N}$ with colors C is a map $f\colon [n_0] \times [n_1] \to C$.

Examples.

Gray levels	$C = \{0, \ldots, 255\}$
RGB colors	$C = \{0, \ldots, 255\}^3$
Real numbers	E.g. $C = \mathbb{R}$ or $C = [0, 1]$
Real tuples	E.g. $C = \mathbb{R}^n$ or $C = [0, 1]^n$

Definition 2. For any digital image $f\colon [n_0] \times [n_1] \to C$, consider the graph $G = (V, E)$ with $V = [n_0] \times [n_1]$ and such that for any $u, v \in V$ we have $\{u, v\} \in E$ if and only if $|u - v| = 1$. It is called the **pixel grid graph** of the image. Its nodes are called the **pixels** of the image.

Definition 3. For any $n_0, n_1 \in \mathbb{N}$ and any set C, a **point operator** on digital images of width n_0, height n_1 and with colors C is a function

$$\varphi\colon C^{[n_0]\times[n_1]} \to C^{[n_0]\times[n_1]} \tag{1}$$

such that there exists a function

$$\chi\colon C \times [n_0] \times [n_1] \to C \tag{2}$$

such that for every digital image $f\colon [n_0] \times [n_1] \to C$ and every pixel $(x, y) \in [n_0] \times [n_1]$, we have

$$\varphi(f)(x, y) = \chi(f(x, y), x, y) \ . \tag{3}$$

Remark. The color $\varphi(f)(x, y)$ of the image $\varphi(f)$ at the pixel (x, y) depends only on the color $f(x, y)$ of the image f at that same location, and on the location (x, y) itself.

Example. Every $\xi\colon C \to C$ defines a point operator $\varphi_\xi\colon f \mapsto \xi \circ f$.

Gamma Operator

Definition 4. Let $C = [0, 1]$. For any $\gamma \in (0, \infty)$ and the function $\xi \colon C \to C : c \mapsto c^\gamma$, the point operator $\varphi_\xi \colon f \mapsto \xi \circ f$ is called the **gamma operator**.

$\gamma = \frac{1}{4}$ $\gamma = \frac{1}{2}$ $\gamma = 1$ $\gamma = 2$ $\gamma = 4$

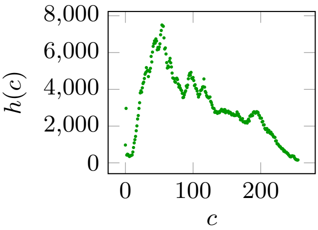
Histogram equilibration

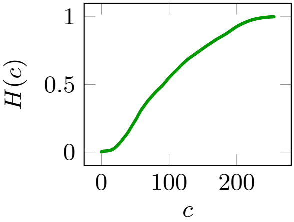
Definition 5. The **histogram** of a digital image $f\colon [n_0] \times [n_1] \to C \subseteq \mathbb{R}$ is the function $h\colon C \to \mathbb{N}_0$ such that for any $c \in C$ we have

$$h(c) = |\{r \in [n_0] \times [n_1] \mid f(r) = c\}| \tag{4}$$

The **cumulative distribution of colors** is the function $H\colon C \to [0,1]$ such that for any $c \in C$ we have

$$H(c) = \frac{1}{n_0\, n_1} \sum_{\substack{c' \in f([n_0]\times[n_1]) \\ c' \leq c}} h(c) \tag{5}$$





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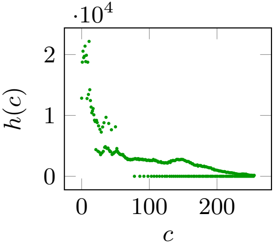
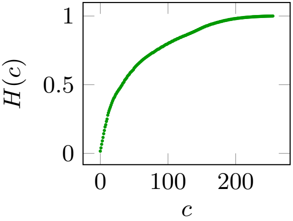




Definition 6. For any $C = [c^-, c^+] \subseteq \mathbb{R}$ and any monotonous function $H\colon C \to [0,1]$ such that $H(c^+) = 1$, H-**equilibration** is the function

$$\begin{aligned} \xi_H\colon \quad [c^-, c^+] &\to [c^-, c^+] \\ c &\mapsto c^- + (c^+ - c^-)\, H(c) \end{aligned}$$

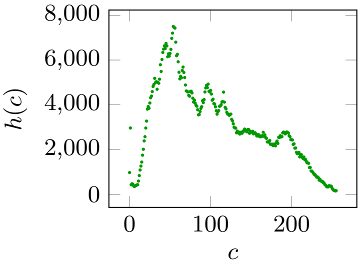
For fixed H and fixed $n_0, n_1 \in \mathbb{N}$, H-equilibration defines a point operator that we call the H-**equilibrator**:

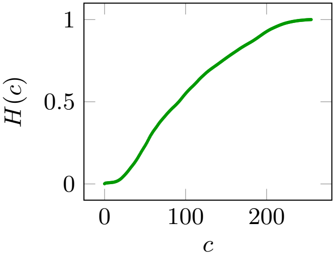
$$\begin{aligned} \varphi_{\xi_H}\colon \quad C^{[n_0]\times[n_1]} &\to C^{[n_0]\times[n_1]} \\ f &\mapsto \xi_H \circ f \end{aligned}$$

For any digital image f with the cumulative distribution H of colors C, we call the image $\varphi_{\xi_H}(f)$ the **self-equilibration of** f.

Question. Is self-equilibration a point operator?

Histogram equilibration





Histogram equilibration